Two-Valued and Three-Valued Integrand Codes for Transmission of Real-Valued Self-Orthogonal Finite-Length Sequence

Yoshihiro Tanada, Peng Shao, and Yuuki Fujii, Non-members

ABSTRACT

A real-valued self-orthogonal finite-length sequence has an impulsive autocorrelation function and is useful for signal transmission but requires the linearity of a transmitting amplifier. This paper proposes two-valued and three-valued integrand codes in order to transmit the real-valued self-orthogonal finite-length sequence. The codes are integrated through a transmission path and leads to those with a chip waveform similar to a one-cycle sine pulse. Correlation output for the three-valued integrand code takes the lower sidelobe error than that for the two-valued integrand code.

Keywords: Finite-Length Sequence, Self-Orthogonal Sequence, Real-Valued Sequence, Two-Valued Integrand Code, Three-Valued Integrand Code, PAM Code

1. INTRODUCTION

A finite-length sequence with a sharp autocorrelation function is effective for a spread spectrum system. A Barker sequence is a two-valued sequence with a low sidelobe autocorrelation function but its length is 13 at most [1]. A real-valued finite-length sequence with a zero sidelobe autocorrelation function except at both shift ends exists for every length and can be called the real-valued selforthogonal or shift-orthogonal finite-length sequence since its shifted sequences are orthogonal. The real-valued selforthogonal finite-length sequences with the different patterns can organize an orthogonal set and have a fast correlation processing algorithm [2], [3]. However, the real-valued self-orthogonal finite-length sequence requires the linearity of a transmitting amplifier.

This paper proposes two-valued and three-valued integrand codes in order to transmit the real-valued self-orthogonal finite-length sequence. The code has the chip waveform of two or three values with a chip duration divided, and is integrated through a transmission path, and leads to a dc-free code with a chip waveform similar to a one-cycle sine pulse. Correlation output for the three-valued integrand code takes the lower sidelobe error than that for the two-valued integrand code.

2. REAL-VALUED FINITE-LENGTH SEQUENCE WITH IMPULSIVE AUTOCORRELATION FUNCTION

A finite-length sequence whose autocorrelation function has no sidelobes except at left and right shift ends is an ideal finite-length PN (pseudonoise) sequence and can be called a self-orthogonal or shift-orthogonal finite-length sequence. An aperiodic autocorrelation function of a complex-valued self-orthogonal finite-length sequence \{a_{M,l,i}\} of length \(M\), member \(l\) and ordinal \(i\) is represented as

\[
\rho_{M,l,i} = \frac{1}{M} \sum_{i=0}^{M-1} a_{M,l,i} a_{M,l,i-i}^{*}
\]

(1)

where \(a_{M,l,i} = 0\) for \(i < 0\) and \(i > M - 1\), and \(\epsilon\) is shift, and \(^{*}\) denotes complex conjugate. \(\epsilon_{M-1}\) is a shift-end complex value given by

\[
\epsilon_{M-1} = |\epsilon_{M-1}| \cdot e^{j\varphi_{M}}
\]

(2)

where \(\varphi_{M}\) is phase and \(j = \sqrt{-1}\). Fig.1 shows an example of the finite-length sequence and its autocorrelation function. The sequence is solved by the aid of a sequence-weighted impulse train arising at every time-chip interval \(T_c\)

\[
a_{M,l}(t) = \sum_{i=0}^{M-1} a_{M,l,i} \delta(t - iT_c)
\]

(3)

and its Fourier transform

\[
A_{M,l}(f) = \sum_{i=0}^{M-1} a_{M,l,i} Z^{-i}
\]

(4)

where \(\delta(t)\) is Dirac’s delta function of time \(t\), and \(Z = e^{j2\pi fT_c}\), and \(f\) is frequency. From Eq.(4), an energy spectrum of \(a_{M,l}(t)\) is connected with the
Fig. 1: (a) Self-orthogonal finite-length sequence. (b) Autocorrelation function.

The autocorrelation function \( \{\rho_{M,l,i}\} \) of the sequence \( \{a_{M,l,i}\} \) is given by

\[
|A_{M,l}(f)|^2 = M \sum_{i=-(M-1)}^{M-1} \rho_{M,l,i} Z^{-i}.
\]

(5)

Substituting Eq. (1) to Eq. (5) and factorizing a polynomial with respect to \( Z^{-1} \) yields a solution for the sequence in spectral domain [2], given by

\[
A_{M,l}(f) = \sqrt{M|\epsilon_{M-1}|} e^{j\phi_M} K_{M,l} \\
\times \prod_{m=0}^{M-2} \left\{ Z^{-1} - \gamma_{M,l,m} e^{-j\frac{2\pi m}{M-1}} e^{j\frac{2\pi m(M+1)}{M-1}} \right\}
\]

where

\[
K_{M,l} = \frac{1}{\sqrt{\prod_{m=0}^{M-2} \gamma_{M,l,m}}}
\]

(7)

\[
\gamma_{M,l,m} = \alpha_M \text{ or } \beta_M
\]

(8)

\[
\alpha_M = \left( 1 + \frac{1 - 4|\epsilon_{M-1}|^2}{2|\epsilon_{M-1}|} \right)^{1/2}
\]

(9)

\[
\beta_M = \frac{1}{\alpha_M}
\]

(10)

Eq. (6) means that the sequence is obtained by convolving \( M^{-1} \) elementary sequences of length 2 and each expansion coefficient of the polynomial corresponds to the sequence value.

If the sequence takes real values, then \( \phi_M = 0 \) or \( \pi \) and a pair of elementary sequences with length 2 and complex conjugate values are combined to a partial sequence with length 3 and real values. Thus we have four formulae for the real-valued sequences according to positive or negative shift-end value of \( \epsilon_{M-1} \) and even or odd length of \( M \) as the followings.

For positive \( \epsilon_{M-1} \) and even \( M \), we have the solution

\[
A_{M,l}(f) = \sqrt{M|\epsilon_{M-1}|} K_{M,l} (Z^{-1} + \gamma_{M,l,o}) \\
\times \prod_{m=1}^{M-2} \left( Z^{-2} + 2\gamma_{M,l,m} Z^{-1} \right) \\
\times \cos \frac{2\pi m}{M-1} + \gamma_{M,l,m}
\]

(11)

by replacing \( m = 0, 1, 2, \ldots, M - 2 \) with \( m + \frac{M-2}{2}, m = 0, \pm 1, \pm 2, \ldots, \pm \frac{M-2}{2} \) in \( e^{j\frac{2\pi m(M+1)}{M-1}} \) of Eq. (6).

For positive \( \epsilon_{M-1} \) and odd \( M \), we have the solution

\[
A_{M,l}(f) = \sqrt{M|\epsilon_{M-1}|} K_{M,l} \\
\times \prod_{m=1}^{M-2} \left( Z^{-2} + 2\gamma_{M,l,m} Z^{-1} \right) \\
\times \cos \left( 2\frac{m\pi}{M-1} + \gamma_{M,l,m} \right)
\]

(12)

by replacing \( m = 0, 1, 2, \ldots, M - 2 \) with \( m + \frac{M-3}{2}, m = \pm 1, \pm 2, \ldots, \pm \frac{M-3}{2} \) in \( e^{j\frac{2\pi m(M+1)}{M-1}} \) of Eq. (6).

For negative \( \epsilon_{M-1} \) and even \( M \), we have the solution

\[
A_{M,l}(f) = -\sqrt{M|\epsilon_{M-1}|} K_{M,l} (Z^{-1} + \gamma_{M,l,o}) \\
\times \prod_{m=1}^{M-2} \left( Z^{-2} - 2\gamma_{M,l,m} Z^{-1} \right) \\
\times \cos \frac{2\pi m}{M-1} + \gamma_{M,l,m}
\]

(13)

by replacing \( m = 0, 1, 2, \ldots, M - 2 \) with \( m + \frac{M-2}{2}, m = 0, 1, 2, \ldots, \frac{M-2}{2} \) in \( e^{j\frac{2\pi m(M+1)}{M-1}} \) of Eq. (6).

For negative \( \epsilon_{M-1} \) and odd \( M \), we have the solution

\[
A_{M,l}(f) = -\sqrt{M|\epsilon_{M-1}|} K_{M,l} (Z^{-1} - \gamma_{M,l,o}) \\
\times \prod_{m=1}^{M-2} \left( Z^{-2} - 2\gamma_{M,l,m} Z^{-1} \right) \\
\times \cos \frac{2\pi m}{M-1} + \gamma_{M,l,m}
\]

(14)

by replacing \( m = 0, 1, 2, \ldots, M - 2 \) with \( m + \frac{M-1}{2}, m = 0, \pm 1, \pm 2, \ldots, \pm \frac{M-1}{2} \) in \( e^{j\frac{2\pi m(M+1)}{M-1}} \) of Eq. (6).

According to the combinations of \( \gamma_{M,l,m} = \alpha_M \) or \( \beta_M \), the groups of the sequences of from Eqs. (11), (12), (13) and (14) have 2\( 2^2 \), 2\( 2^3 \), and 2\( 2^4 \) members, respectively.

Tables 1 and 2 show numerical examples of the sequences of length \( M = 5 \) and shift-end values \( \epsilon_4 = -1/5, 1/5 \), respectively. When \( M = 5, \epsilon_4 = -1/5 \), from Eq. (14), we obtain the spectrum

\[
A_{5,6}(f) = -\sqrt{5|\epsilon_4|} K_{5,6} \\
\times (Z^{-1} - \alpha_5)(Z^{-1} + \beta_5)(Z^{-2} + \beta_5^2) \\
= -\alpha_5(Z^{-2} - (\alpha_5 - \beta_5)Z^{-1} - 1)(Z^{-2} + \beta_5^2) \\
= -\alpha_5 Z^{-4} + \alpha_5(\alpha_5 - \beta_5) Z^{-3} + (\alpha_5 - \beta_5) Z^{-2} + \beta_5(\alpha_5 - \beta_5) Z^{-1} + \beta_5 
\]

(15)

and then the sequence

\[
[\alpha_{5,6,i}] = [\beta_5, \beta_5(\alpha_5 - \beta_5), \alpha_5 - \beta_5, \alpha_5(\alpha_5 - \beta_5), -\alpha_5]
\]

(16)
where the member number \( l \) is defined by the binary notation \((110)_2 = 6\) corresponding to the reversed product \((Z^{-2} + \beta_2)(Z^{-1} + \beta_3)(Z^{-1} - \alpha_3)\). In other case, the member number \( l \) is defined similarly.

Table 1: Sequences \( \{a_{M,l,i}\}(M=5, \varepsilon_4 = -1/5) \).

<table>
<thead>
<tr>
<th>( l )</th>
<th>( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(001)_2 = 1</td>
<td>1.4795</td>
<td>-1.1889</td>
<td>-0.8036</td>
<td>-0.5432</td>
<td>-0.6759</td>
<td></td>
</tr>
<tr>
<td>(010)_2 = 2</td>
<td>1.4795</td>
<td>1.1889</td>
<td>-0.8036</td>
<td>+0.5432</td>
<td>-0.6759</td>
<td></td>
</tr>
<tr>
<td>(011)_2 = 3</td>
<td>0.6759</td>
<td>-0.5432</td>
<td>0.8036</td>
<td>-1.1889</td>
<td>-1.4795</td>
<td></td>
</tr>
<tr>
<td>(110)_2 = 6</td>
<td>0.6759</td>
<td>0.5432</td>
<td>0.8036</td>
<td>1.1889</td>
<td>-1.4795</td>
<td></td>
</tr>
</tbody>
</table>

3. PAM CODE FOR THE SEQUENCE AND ITS AUTOCORRELATION FUNCTION

In order to apply the real-valued sequence to a spread spectrum system, the real-valued sequence \( \{a_{M,l,i}\} \) is approximated by an integer sequence \( \{\hat{a}_{M,l,i}\} \) as follows:

\[
a_{M,l,i} = K_c \hat{a}_{M,l,i}
\]

where \( K_c \) is a power-normalization coefficient, and a spreading PAM (pulse amplitude modulation) code of time-dependent function is constructed by

\[
\tilde{a}_{M,l}(t) = w_c(t) \otimes a_{M,l}(t) = K_c w_c(t) \otimes \hat{a}_{M,l}
\]

where \( w_c(t) \) is an isolated chip waveform of time \( t \), and \( \otimes \) denotes a convolution in time domain as

\[
w_c(t) \otimes a_{M,l}(t) = \int_{-\infty}^{\infty} w_c(\tau) a_{M,l}(t-\tau) d\tau
\]

and the impulse train weighted by \( \{\hat{a}_{M,l,i}\} \) is obtained co-re-sponding to Eq. (3) as

\[
\hat{a}_{M,l}(t) = \sum_{i=0}^{M-1} \hat{a}_{M,l,i} \delta(t - iT_c).
\]

Fig. 2 shows waveforms concerning Eq. (18), when a chip waveform is a unit block pulse of duration \( T_c \).

An aperiodic autocorrelation function of a code waveform of Eq. (18) is given by

\[
\varphi_{M,l}(\tau) = \frac{1}{T_c} \int_{-\infty}^{\infty} \tilde{a}_{M,l}(t) \tilde{a}_{M,l}(t-\tau) dt
\]

\[
= \frac{1}{T_c} \hat{a}_{M,l}(\tau) \otimes \hat{a}_{M,l}(-\tau)
\]

\[
= \psi_{c,e}(\tau) \otimes \rho_{M,l,l}(\tau)
\]

where \( \tau \) is time shift, and \( T = MT_c \), and an autocorrelation function \( \psi_{c,e}(\tau) \) of the chip waveform \( w_c(t) \) is represented by

\[
\psi_{c,e}(\tau) = \frac{1}{T_c} \int_{-\infty}^{\infty} w_c(t) w_c(t-\tau) dt
\]

and the impulse train weighted by \( \{\rho_{M,l,l,i}\} \) is expressed as

\[
\rho_{M,l,l,i}(\tau) = \sum_{i=0}^{M-1} \rho_{M,l,l,i} \delta(\tau - iT_c)
\]

Eq.(22) explains that the autocorrelation function of a code waveform is given by a convolution between the autocorrelation function of a chip waveform and the impulse train weighted by the autocorrelation function of a sequence. Fig.3 shows the waveforms concerning the autocorrelation functions of the waveforms in Fig.2.

When a spreading code is constructed from the approximated integer sequence \( \{\hat{a}_{M,l,i}\} \), the approximated real-valued sequence is expressed from Eq.(17) as

\[
K_c \hat{a}_{M,l,i} = a_{M,l,i} + \Delta a_{M,l,i}
\]

where \( \{\Delta a_{M,l,i}\} \) is an error sequence deviated from the true sequence \( \{a_{M,l,i}\} \). The autocorrelation func-
tion of the approximated sequence becomes
\[
\frac{1}{M} \sum_{i=0}^{M-1} \left( a_{M,i} + \Delta a_{M,i} \right) \cdot \left( a_{M,i-i} + \Delta a_{M,i-i} \right)
\]
\[
= \rho_{M,i,i} + \frac{1}{M} \sum_{i=0}^{M-1} \left( \Delta a_{M,i} \cdot a_{M,i-i} + \Delta a_{M,i} \cdot \Delta a_{M,i-i} \right).
\] (26)

Suppose the error \( \{ \Delta a_{M,i} \} \) is independent random number with zero mean and variance \( \sigma^2 \). The variance of the first term of the random parts in Eq. (26) is given by
\[
\frac{1}{M^2} \sum_{i_1=0}^{M-1} \sum_{i_2=0}^{M-1} E[\Delta a_{M,i_1} \Delta a_{M,i_2}]
\]
\[
\times a_{M,i_1-i_2} a_{M,i_2-i} = \frac{\sigma^2}{M^2} \sum_{i=0}^{M-1} a_{M,i-i}^2.
\] (27)

The third term of those parts is an autocorrelation function of finite-length random number, given by
\[
\frac{1}{M^2} E \left( \sum_{i=0}^{M-1} a_{M,i} \Delta a_{M,i-i} \right)^2
\]
\[
= \frac{\sigma^2}{M^2} \sum_{i=0}^{M-1} a_{M,i+i}^2.
\] (28)

The third term of those parts is an autocorrelation function of finite-length random number, given by
\[
\frac{1}{M^2} \sum_{i=0}^{M-1} E[\Delta a_{M,i} \Delta a_{M,i-i}] = \begin{cases} \sigma^2, & i = 0 \\ 0, & i \neq 0. \end{cases}
\] (29)

The variances of Eqs. (27) and (28) have the maximum value \( \sigma^2/M \) at shift \( i = 0 \) and decrease as shift goes away from the zero shift. The autocorrelation function of Eq. (29) takes the value \( \sigma^2 \) only at shift \( i = 0 \). If we use a reference sequence with an error different from that of a transmitted sequence in a spread spectrum system, the third term of the random parts of Eq. (26) is replaced by a crosscorrelation function between finite-length random numbers, which take zero at all shift. Then, the magnitude of the variances corresponding to Eqs. (27) and (28) are different from each other. Whichever reference sequence is used, the sidelobe error of the autocorrelation function decreases as the length \( M \) increases.

4. INTEGRAND CODE FOR THE SEQUENCE AND ITS TRANSMISSION

In a practical spread spectrum system, we are apt to encounter the problems on electronic device, such as the linearity deficiency of a transmitting amplifier and the structural simplicity of a correlator. Two-valued or three-valued sequence would be effective to solve these problems. A two-valued PWM (pulse width modulation) code is mostly considered for the real-valued sequence, but the rate of the clock pulse to generate the code becomes so much high in proportion to the division number of a chip duration \( T_c \), such that the division number is 64 or the more in a 6-bit quantization.

Therefore, we propose the two-valued and three-valued integrand codes which do not require the high-rate clock pulse [4], [5]. These codes are supposed to pass through a transmitting amplifier, a passive low pass filter and a transmitting antenna. Fig.4 illustrates the composition of a two-valued integrand code and its integration. One chip time duration \( T_c \) is divided into 8 time slots and the 8-point moving average is used instead of an integration. (a), (b), (c), (d), (e), (f), (g), and (h) at the left half in Fig.4 are the code components which produce the respective front area 7, 5, 3, 1, -7, -5, -3, -1 after the moving average. The value of the front area in the 8 division slots is given by
\[
N_b = 7d_3 + 5d_2 + 3d_1 + d_0
\] (30)

where \( d_3, d_2, d_1, d_0 \in \{ +1, -1 \} \). In this case, the value \( N_b/2 \) takes 0, \( \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 7 \) and \( \pm 8 \), and the original real values to be mapped into \( \pm 6 \) is mapped into \( \pm 5 \) or \( \pm 7 \) so as to reduce the error, which decreases as the division number increases. The waveform after repeating the moving average for the multiple time resembles a one-cycle sine pulse, whose crest level and front area are proportional to the original real value. The above moving average corresponds to a low pass filter of the following impulse response:
\[
h(t) = \frac{1}{T_c} \{ u(t) - u(t - T_c) \}
\] (31)

where \( u(t) \) is a unit step function. Since the multiple moving average roughly corresponds to a low pass
filter of a Gaussian impulse response and produces the analogous waveform at every chip timing, the PAM code with the chip waveform \( w_c(t) \) similar to a one-cycle sine pulse leads to be generated through a transmission path.

In the two-valued integrand code, when the different two or more codes in each time interval of \( T_c \) lead to the same integrated value, the code producing the waveform with the best approximation is selected. Generally, the two-valued integrand code with even division number \( n \) takes the level number \( \lambda_b \approx n^2 \frac{n}{4} + 1 \) (32) where the division numbers \( n = 8, 16, 24 \) correspond to the quantization bit numbers 4, 6, 7 respectively.

On the other hand, three-valued integrand code leads the value as

\[
N_t = 7d_3 + 5d_2 + 3d_1 + d_0 \quad (33)
\]

where \( d_3 + d_2 + d_1 + d_0 \in \{+1, 0, -1\} \), and takes the level number for even division number \( n \)

\[
\lambda_t = \frac{n^2}{2} + 1 \quad (34)
\]

where the division numbers \( n = 8, 16, 24 \) correspond to the quantization bit numbers 5, 7, 8, respectively.

Table 3 shows the integer sequence \( \{a_{5,1,i}\} \) reduced from the two-valued and three-valued integrand codes of the division numbers \( n = 8, 16 \) for the sequence \( \{a_{5,1,i}( \epsilon_4 = -1/5)\} \) in Table 1.

### Table 3: Integer sequence \( \{\hat{a}_{5,1,i}\} \) for real-valued sequence.

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>From 2-valued code</td>
<td>( n=8 )</td>
<td>8</td>
<td>-6</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>( n=16 )</td>
<td>32</td>
<td>-26</td>
<td>-17</td>
<td>-12</td>
</tr>
<tr>
<td>From 3-valued code</td>
<td>( n=8 )</td>
<td>16</td>
<td>-13</td>
<td>-9</td>
<td>-6</td>
</tr>
<tr>
<td></td>
<td>( n=16 )</td>
<td>64</td>
<td>-51</td>
<td>-35</td>
<td>-23</td>
</tr>
</tbody>
</table>

### 5. NUMERICAL CORRELATION SIMULATION FOR FINITE-LENGTH INTEGRAND CODES

We assume a model of a code transmission system of Fig.5. We evaluate the two-valued and three-valued integrand codes on this model. The codes are constructed from a real-valued finite-length sequence \( \{a_{M,l,i}\} \) of length \( M = 65 \) and shift-end value \( \epsilon_{64} = 1/65 \) where \( |a_{M,l,i}| < 2 \) and \( l = 99999970 = (5F5E0E2)_{16} \). Integer sequence \( \{\hat{a}_{M,l,i}\} \) is generated so that the maximum magnitude of \( \frac{N_b}{2} \) from Eq. (30) or \( N_t \) of Eq. (33) may correspond to the maximum magnitude \( |a_{M,l,i}|_{\text{max}} \) of \( \{a_{M,l,i}\} \). The LPF (low pass filter) is replaced by a moving averager. Since the integrand code leads to a one-cycle sine pulse, we use the chip waveform of Fig.6 (a), and the equivalent dc-free spreading code of Fig.6 (b) as the reference code.

First, in order to examine the effect of the quantization based on the division number \( n \), we transmit the PAM code of the same code as the reference code. Each code data is generated by \( 2^n \) points per chip interval \( T_c \). The moving average equivalent to the impulse response of Eq. (31) is repeated at 5 times in this case. Figs.7 and 8 show the correlation outputs in linear scale and in logarithmic scale, for the PAM codes from the two-valued and three-valued integrand codes, respectively. These results imply that the three-valued integrand code gives the higher accuracy than the two-valued integrand code.

Next, in order to examine the effect of the analogous chip waveforms, we transmit the three-valued integrand code of the division number \( n = 24 \) and varies

### Fig.5: Model of code transmission system.

### Fig.6: (a) Chip waveform. (b) Equivalent spreading code.

### Fig.7: Correlation outputs for PAM codes from two-valued integrand codes. (a) Division number \( n = 8 \). Division number \( n = 16 \).
the multiplicity of the moving average. Fig. 9 shows the correlation outputs through the moving average of the multiplicity number 2, 4 and 6, where the moving average equivalent to the impulse response of Eq. (31) is repeated according to the multiplicity number. These results explain that the multiplicity number 4 is sufficient but the more effective moving average is desired to suppress sidelobes and mainlobe pulse width.

Then, through the moving averager of the multiplicity number 4, we transmit the three-valued integrand codes of the division number \( n = 8, 16 \) and 24. Fig. 10 shows the correlation outputs for these codes. These results explain that the correlation output sidelobes decrease as the division number increases though they are disturbed by the waveform error at every chip timing.

**Table 4:** Correlation output levels in decibel for integrand codes to those for PAM codes with chip waveform duration \( 2T_c \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>8</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-valued code</td>
<td>-4.76</td>
<td>-4.64</td>
<td>-4.60</td>
</tr>
<tr>
<td>Three-valued code</td>
<td>-4.82</td>
<td>-4.54</td>
<td>-4.56</td>
</tr>
</tbody>
</table>

**Table 5:** Correlation output levels in decibel for integrand codes to those for PAM codes with chip waveform duration \( T_c \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>8</th>
<th>16</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-valued code</td>
<td>-0.26</td>
<td>-0.24</td>
<td>-0.18</td>
</tr>
<tr>
<td>Three-valued code</td>
<td>-0.25</td>
<td>-0.15</td>
<td>-0.11</td>
</tr>
</tbody>
</table>

Finally, we examine the magnitude of the correlation outputs when the transmitting amplifier yields the same magnitude of the PAM code and the in-

**Fig. 8:** Correlation outputs for PAM codes from three-valued integrand codes. (a) Division number \( n = 8 \). Division number \( n = 16 \).

**Fig. 9:** Correlation outputs for three-valued integrand code of division number 24. (a) 2-time moving average. (b) 4-time moving average. (c) 6-time moving average.

**Fig. 10:** Correlation outputs for three-valued integrand codes. (a) Division number \( n = 8 \). (b) Division number \( n = 16 \). (c) Division number \( n = 24 \).
tegrand code. We use two PAM codes: one has the chip waveform of Fig.6(a), and the other has its shrunk waveform into duration $T_c$. Tables 4 and 5 show the correlation output levels for the two-valued and three-valued integrand codes when the correlation output level for the PAM code is normalized to $0\,\text{dB}$. Passing through the LPF, the chip waveform with duration $2T_c$ makes the magnitude of sine-like pulse about two times as much as that with duration $T_c$. The correlator extracts a component of the chip waveform with duration $2T_c$. As a result, the PAM codes with chip durations $2T_c$ and $T_c$ produce the correlator output levels about $2.6\,\text{dB}$ and $0.2\,\text{dB}$ higher, respectively, than the integrand codes. The PAM codes with chip duration $2T_c$ produce the wider correlator output than that with chip duration $T_c$. Therefore, the PAM code with chip duration $T_c$ is effective for the practical system. In this case, the integrand code produces the correlator output only $0.2\,\text{dB}$ lower than the PAM code. If the linearity of the transmitting amplifier is not satisfied, the integrand code gains more advantage than the PAM code.

In these simulations, we use the reference code from the integer sequence $\{a_{M,l,i}\}$. If we use the reference code from the real-valued sequence $\{a_{M,l,i}\}$, the sidelobe errors of the correlator outputs become $3\,\text{dB}$ lower.

6. CONCLUSION

The real-valued self-orthogonal finite-length sequence has an impulsive autocorrelation function but requires the linearity of a transmitting amplifier. Hence, we propose the two-valued and three-valued integrand codes and investigate these codes by the numerical simulation. The codes are integrated by the low pass filters to produce the chip waveforms similar to one-cycle sine pulse. The integrand code has the chip waveform of two or three values with a chip duration divided. The correlation outputs for both integrand codes take the sidelobe error which decreases as the division number increase, and the correlation output for the three-valued integrand code takes the lower sidelobe error than that for the two-valued integrand code. Under the amplitude limitation of the transmitting amplifier, both integrand codes present the correlation outputs with the level only lower than the PAM code. The integrand code can gain the higher correlation output than the PAM code if the amplitude of the PAM code is less by the unsatisfied linearity of the transmitting amplifier. The other subjects are to be investigated on the low pass filter design, the analysis of signal to noise ratio, etc. for the future.

ACKNOWLEDGMENT

The authors acknowledge the former graduate student, Mr. Akira Hirosawa for his helpful experiments.

References


Yoshihiro Tanada received the B.E. degree in Electrical Engineering from Okayama University, Okayama, Japan, in 1966, and Ph.D. degree in Electronics from Kyoto University, Kyoto, Japan, in 1979. In 1966, he joined Toyo Communication Equipment Corp., Kawasaki, Japan. From1969 to1988, he was with the Department of Electronics, Okayama University, Okayama, Japan. In 1988, he became an Associate Professor of the Department of Electronics, and in 1991 a Professor of the Department of Information and Computer Science, respectively, Kagoshima University, Kagoshima, Japan. Since 1997, he has been a Professor of the Department of Computer Science and Systems Engineering, Yamaguchi University, Ube, Japan. He received the Best Paper Award from the IEEE in International Symposium on Communication and Information Technologies 2004. His current research interests include orthogonal pseudonoise sequence, spread spectrum system and digital watermark. He is a member of the IEEE , the IEICE of Japan, the IEE of Japan, the ITE of Japan, and the SITA of Japan.

Peng Shao received the B.E. degree in Information Science and Engineering from Shandong University, Jinan, P.R.China, in 2002 , and the M.S. degree in Computer Science and Systems Engineering from Yamaguchi University, Ube, Japan, in 2004. He received the Best Paper Award from the IEEE in International Symposium on Communication and Information Technologies 2004. He is currently a doctoral candidate in the Graduate School of Science and Engineering, Yamaguchi University, Ube, Japan. His research interests include pseudonoise sequence and CDMA communication systems. He is a student member of the IEICE of Japan.
Yuuki Fujii received the B.E. degree in Computer Science and Systems Engineering from Yamaguchi University, Ube, Japan, in 2004. He received the Best Paper Award from the IEEE in International Symposium on Communication and Information Technologies 2004. He is currently a master course student in the Graduate School of Science and Engineering, Yamaguchi University, Ube, Japan. His research interest is CDMA communication systems. He is a student member of the IEICE of Japan.