A Design Method for Smith Predictor for Minimum Phase Time-Delay Plants with Multiple Time-delays Using The Parameterization And The Model Matching Method

Kou Yamada\(^1\) and Hiroshi Takenaga\(^2\), Non-members

ABSTRACT

In this paper, we examine a design method for modified Smith predictor for minimum-phase time-delay plants with multiple time-delays. The modified Smith predictor is well known as an effective time-delay compensator for a plant with large time-delay, and several papers on the modified Smith predictor have been published. The parameterization of all stabilizing modified Smith predictors for minimum phase time-delay plants is obtained by Yamada and Matsushima. However, they do not examine the parameterization of all stabilizing modified Smith predictors for systems with multiple time-delays. The purpose of this paper is to expand the result by Yamada and Matsushima and to propose the parameterization of all stabilizing modified Smith predictors for minimum phase time-delay plants with multiple time-delays. Next, we present a design method of modified Smith predictor for minimum phase time-delay plants with multiple time-delays using the obtained parameterization and the model matching method.

Keywords: Minimum-Phase System, Time-Delay System, Multiple Time-Delays, Smith Predictor, parameterization, Model Matching Method

1. INTRODUCTION

In this paper, we examine a design method for Smith predictors for minimum-phase time-delay plants with multiple time-delays. The Smith predictor is proposed by Smith to overcome time-delay \cite{1} and known as an effective time-delay compensator for a stable plant with large time-delay \cite{1–12}.\footnote{Manuscript received on July 30, 2006; revised on November 5, 2006.} The Smith predictor in \cite{1} cannot be used for plants having an integral mode, because a step disturbance will result in a steady state error \cite{2–4}. To overcome this problem, Watanabe and Ito \cite{4}, Astrom, Hang and Lim \cite{9}, and Matusek and Micic \cite{10} proposed a design method for a modified Smith predictor for time-delay plants with an integrator. Watanabe and Sato expanded the result in \cite{4} and proposed a design method for modified Smith predictors for multivariable systems with multiple time-delays in inputs and outputs \cite{5}.

Because the modified Smith predictor cannot be used for unstable plants \cite{2–11}, De Paor \cite{6}, De Paor and Egan \cite{8} and Kwak, Sung, Lee and Park \cite{12} proposed a design method for modified Smith predictors for unstable plants. Thus, several design methods of modified Smith predictors have been published.

On the other hand, another important control problem is the parameterization problem, the problem of finding all stabilizing controllers for a plant \cite{13–21}. The parameterization of all stabilizing controllers for time-delay plants was considered in \cite{20, 21}, but that of all stabilizing modified Smith predictors was not obtained. Yamada and Matsushima gave the parameterization of all stabilizing modified Smith predictors \cite{22}. Since the parameterization of all stabilizing modified Smith predictors was obtained, we could express previous studies of modified Smith predictors in a uniform manner and could design modified Smith predictors systematically. However, the parameterization in \cite{22} cannot apply for minimum phase time-delay plants with multiple time-delays.

The purpose of this paper is to expand the result in \cite{22} and to propose the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants with multiple time-delays. First, the structure and necessary characteristics of modified Smith predictors are defined. Next, the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants with multiple time-delays is proposed, for both stable and unstable plants. Next, we present a design method of modified Smith predictor to specify input-output characteristic using the fusion of the obtained parameterization and the model matching method. This paper is organized as follows: In Section 2., the Smith predictor is introduced briefly and the problem considered in this paper is explained. In Section 3. and Section 4., the parameterizations of all stabilizing modified Smith predictors for stable and unstable plants with...
2 time-delays are given, respectively. In Section 3. and Section 4., we also clarify the control characteristics using the parameterization of all stabilizing modified Smith predictors. In Section 5., we present a design method of modified Smith predictors to specify input-output characteristic. In Section 6., we expand the result in Section 3. and Section 4. and propose the parameterization of all stabilizing modified Smith predictors for the plant with N time-delays. Simple numerical example is illustrated in Section 7.

\[ G_2(s)e^{-sT_2} \] is decided by the form:

\[ C(s) = \frac{C_1(s)}{1 + C_2(s)e^{-sT_1} + C_3(s)e^{-sT_2}}, \] (1)

where \( C_1(s) \in R(s), C_2(s) \in R(s) \) and \( C_3(s) \in R(s) \). In addition, using the modified Smith predictor in \([1–12]\), the transfer function from \( r \) to \( y \) of the control system in Fig. 1, written as

\[ y = \frac{C(s) (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + C(s) (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} r, \] (2)

has a finite number of poles. That is, the transfer function from \( r \) to \( y \) of the control system in Fig. 1 is written as

\[ y = (\hat{G}_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) r, \] (3)

where \( \hat{G}_1(s) \in RH_\infty(i = 1, 2) \). Therefore, we call \( C(s) \) the modified Smith predictor if \( C(s) \) takes the form of (1) and the transfer function from \( r \) to \( y \) of the control system in Fig. 1 has a finite number of poles.

The problem considered in this paper is to obtain the parameterization of all modified Smith predictors \( C(s) \) that make the control system in Fig. 1 stable. In Section 3., we propose the parameterization of all stabilizing modified Smith predictors \( C(s) \) for stable plants. In Section 4., we expand the result in Section 3. and propose the parameterization of all stabilizing modified Smith predictors \( C(s) \) for unstable plants.

### 3. THE PARAMETERIZATION FOR STABLE PLANTS

The parameterization of all stabilizing modified Smith predictors for the stable plant \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is summarized in the following theorem.

**Theorem 1:** \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is assumed to be stable. The parameterization of all stabilizing modified Smith predictors \( C(s) \) takes the form

\[ C(s) = 1 - Q(s) (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}), \] (4)

where \( Q(s) \in RH_\infty \) is any function.

**Proof:** First, the necessity is shown. That is, if the controller \( C(s) \) in (1) makes the control system in Fig. 1 stable and makes the transfer function from \( r \) to \( y \) of the control system in Fig. 1 have a finite number of poles, then \( C(s) \) takes the form of (4). From the assumption that the controller \( C(s) \) in (1) makes the transfer function from \( r \) to \( y \) of the control system in Fig. 1 have a finite number of poles,

\[ C(s) \left( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \right) \]

\[ \frac{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{\left[ 1 + (G_1(s) + C_1(s)G_1(s)) \right] e^{-sT_1}} \]

\[ + (C_3(s) + C_1(s)G_2(s)) e^{-sT_2} \] (5)
has a finite number of poles. This implies that
\[ C_2(s) = -C_1(s)G_1(s) \]  
(6)
and
\[ C_3(s) = -C_1(s)G_2(s) \]  
(7)
are necessary, that is:
\[ C(s) = \frac{C_1(s)}{1 - C_1(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}. \]  
(8)

From the assumption that \( C(s) \) in (1) makes the control system in Fig. 1 stable, \( C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}/(1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}) \), \( C(s)/\{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}\} \), \( (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})/(1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}\) and \( 1/(1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}) \) are stable. From simple manipulation, (6) and (7), we have
\[
\frac{C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = C_1(s) \]  
(9)
\[
\frac{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = \left(1 - C_1(s)G_1(s)e^{-sT_1} - C_1(s)G_2(s)e^{-sT_2}\right) 
\cdot \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \]  
(11)
and
\[
\frac{1}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = 1 - C_1(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}. \]  
(12)

It is obvious that the necessary condition for all the transfer functions in (9), (10), (11) and (12) to be stable is \( C_1(s) \in RH_{\infty} \). Using \( Q(s) \in RH_{\infty} \), let \( C_1(s) \) be
\[ C_1(s) = Q(s), \]  
(13)
we find that \( C(s) \) takes the form of (4). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if \( C(s) \) takes the form of (4) and \( Q(s) \in RH_{\infty} \), then the controller \( C(s) \) makes the control system in Fig. 1 stable and makes the transfer function from \( r \) to \( y \) of the control system in Fig. 1 have a finite number of poles. From simple manipulation, we have
\[
\frac{C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = Q(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}, \]  
(14)
\[
\frac{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = \left\{1 - Q(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}\right\} 
\cdot \left(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\right) \]  
(16)
and
\[
\frac{1}{1 + C(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}} = 1 - Q(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}. \]  
(17)

From the assumption that \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is stable and \( Q(s) \in RH_{\infty} \), (14), (15), (16) and (17) are all stable. In addition, because the transfer function from \( r \) to \( y \) of the control system in Fig. 1 takes the form (14) and \( Q \in RH_{\infty} \), the transfer function from \( r \) to \( y \) of the control system in Fig. 1 has a finite number of poles.

We have thus proved Theorem 1.

**Note 1:** Note that because the proof of Theorem 1 does not require the assumption that \( G_1(s) + G_2(s)e^{-s(T_2-T_1)} \) is of minimum phase. Therefore even if \( G_1(s) + G_2(s)e^{-s(T_2-T_1)} \) is of non-minimum phase, the parameterization of all stabilizing modified Smith predictors for stable plant \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is given by Theorem 1.

Next, we explain the control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors in (4). The transfer function from the reference input \( r \) to the output \( y \) of the control system in Fig. 1 takes the form
\[ y = Q(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\} r. \]  
(18)

Therefore, for the output \( y \) to follow the step reference input \( r = 1/s \) without steady state error,
\[ Q(0)(G_1(0) + G_2(0)) = 1 \]  
(19)
must be satisfied.

The disturbance attenuation characteristics are as follows. The transfer function from the disturbance \( d \) to the output \( y \) of the control system in Fig. 1 is given by
\[ y = \{1 - Q(s)\{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}\}\} d. \]  
(20)
Therefore, to attenuate the step disturbance \( d = 1/s \) effectively, \( Q(s) \) must satisfy
\[ Q(0)(G_1(0) + G_2(0)) = 1. \]  
(21)

4. THE PARAMETERIZATION FOR UNSTABLE PLANTS

In this section, we expand the result in Section 3, and propose the parameterization of all stabilizing modified Smith predictors \( C(s) \) for unstable minimum phase plants.
This parameterization is summarized in the following theorem.

**Theorem 2:** It is assumed that \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is unstable and \( G_1(s) + G_2(s)e^{-s(T_2-T_1)} \) is of minimum phase. For simplicity, the unstable poles of \( G_1(s) \) and that of \( G_2(s) \) are assumed to be distinct. That is, when \( s_{1i}(i = 1, \ldots, n_1) \) denote unstable poles of \( G_1(s) \), \( s_{1j} \neq s_{1j} \) \( (i \neq j; i = 1, \ldots, n_1, j = 1, \ldots, n_1) \). In like manner, when \( s_{2i}(i = 1, \ldots, n_2) \) denote unstable poles of \( G_2(s) \), \( s_{2i} \neq s_{2j} \) \( (i \neq j; i = 1, \ldots, n_2, j = 1, \ldots, n_2) \), \( s_{1i} \neq s_{2j} \) \( (i = 1, \ldots, n_1; j = 1, \ldots, n_2) \). Under these assumptions, there exists \( G_u(s) \in \mathcal{U} \) satisfying

\[
G_u(s) = \frac{G_u(s_{1i})}{G_{s1}(s_{1i})e^{-s_{1i}T_1}} \quad (22)
\]

and

\[
\bar{G}_u(s_{2i}) = \frac{G_u(s_{2i})}{G_{s2}(s_{2i})e^{-s_{2i}T_2}} \quad (23)
\]

where \( G_u(s)(i = 1, 2) \) is a stable function of \( G_i(s)(i = 1, 2) \), that is, when \( G_i(s)(i = 1, 2) \) is factorized as

\[
G_i(s) = G_u(s)G_{si}(s), \quad (24)
\]

\( G_u(s) \) is the unstable biproper minimum phase function and \( G_{si}(s) \) is the stable function. Using these functions, the parameterization of all stabilizing modified Smith predictors \( C(s) \) is written as

\[
C(s) = \frac{C_f(s)}{1 - C_f(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}, \quad (25)
\]

where \( C_f(s) \) is given by

\[
C_f(s) = \frac{G_u(s)}{G_{s1}(s)G_{u2}(s)} \left(1 + \frac{Q(s)}{G_u(s)G_{u2}(s)} \right) \quad (26)
\]

and \( Q(s) \in RH_\infty \) is any function.

The proof of Theorem 2 requires the following Lemma.

**Lemma 1:** It is assumed that \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is unstable and \( G_1(s) + G_2(s)e^{-s(T_2-T_1)} \) is of minimum phase. For simplicity, the unstable poles \( s_{1j}(i = 1, 2; j = 1, \ldots, n_i) \) of \( G(s) \) are assumed to be distinct. Under these assumptions, there exists \( G_{si}(s) \in \mathcal{U} \) satisfying \( (22) \) and \( (23) \), where \( G_{si}(s) \) is the stable function of \( G_i(s) \).

**Proof:** \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) is rewritten by

\[
G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} = G_{s1}(s)G_{u2}(s)e^{-sT_1} \cdot \left( \frac{G_{s1}(s)}{G_{u2}(s)} + \frac{G_{s2}(s)}{G_{u1}(s)}e^{-s(T_2-T_1)} \right). \quad (27)
\]

From the assumption that \( G_1(s) + G_2(s)e^{-s(T_2-T_1)} \) is of minimum phase, \( 1/G_{u1}(s_{1i}) = 0(i = 1, \ldots, n_1) \) and \( 1/G_{u2}(s_{2j}) = 0(i = 1, \ldots, n_2) \), for all \( s_{1i}(i = 1, \ldots, n_1) \) and \( s_{2j}(i = 1, \ldots, n_2) \) on the real axis,

\[
\begin{align*}
\left\{ G_{s1}(s) + G_{s2}(s)e^{-s(T_2-T_1)} \right\}G_{u1}(s) &= \frac{G_{s1}(s)}{G_{u2}(s_{2i})} \quad (28) \\
\left\{ G_{s1}(s) + G_{s2}(s)e^{-s(T_2-T_1)} \right\}G_{u1}(s) &= \frac{G_{s2}(s_{2i})e^{-s(T_2-T_1)}}{G_{u1}(s_{2i})} \quad (29)
\end{align*}
\]

are all the same sign. Since \( e^{-s_{1i}T_1} > 0 \) and \( e^{-s_{2j}T_2} > 0 \) hold true for all \( s_{1i}(i = 1, \ldots, n_1) \) and \( s_{2j}(i = 1, \ldots, n_2) \) on the real axis, \( (28) \) and \( (29) \) implies that \( G_{u1}(s_{2i}) = G_{u1}(s_{1i})e^{-s_{1i}T_1} \) and \( G_{u2}(s_{2i}) \) are the same sign for all \( s_{1i}(i = 1, \ldots, n_1) \) and \( s_{2j}(i = 1, \ldots, n_2) \) on the real axis. From Theorem 2.3.3 in [16], there exists \( G_u(s) \in \mathcal{U} \) satisfying \( (22) \) and \( (23) \). We have thus proved Lemma 1.

Using this Lemma, we shall show the proof of Theorem 2.

**Proof:** First, the necessity is shown. If the controller \( C(s) \) in (1) makes the control system in Fig. 1 stable and makes the transfer function from \( r \) to \( y \) of the control system in Fig. 1 have a finite number of poles, then \( C(s) \) takes the form (25). From the same assumption,

\[
C(s) = \frac{C_f(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + C_f(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} = \frac{C_f(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + (C_2(s) + C_1(s)G_1(s))e^{-sT_1}} + (C_3(s) + C_1(s)G_2(s))e^{-sT_2} \quad (30)
\]

has a finite number of poles. This implies that

\[
C_2(s) = -C_1(s)G_1(s) \quad (31)
\]

and

\[
C_3(s) = -C_1(s)G_2(s) \quad (32)
\]

are satisfied, that is, \( C(s) \) is necessarily

\[
C(s) = \frac{C_1(s)}{1 - C_1(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}. \quad (33)
\]

From the assumption that \( C(s) \) in (1) makes the control system in Fig. 1 stable, \( C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}/(1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})) \), \( C(s)/[1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})] \), \( (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})/(1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})) \) and \( 1/[1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})] \) make the transfer function from \( r \) to \( y \) of the control system in Fig. 1 have a finite number of poles. The proof of Theorem 2 is thus completed.
satisfying (31) and (32), we have

\[
\frac{C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} = C_1(s),
\]

(34)

\[
\frac{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} = 1 - C_1(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \cdot \left(1 + \frac{C_1(s) - G_u(s)}{G_u(s)}\right),
\]

(36)

and

\[
1 + \frac{C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} = 1 - C_1(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \cdot \left(1 + \frac{C_1(s) - G_u(s)}{G_u(s)}\right).
\]

(37)

From the assumption that \(C_1(s)\) and \(C_1(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})\) are stable, \(C_1(s)\) takes the form

\[
C_1(s) = \frac{\hat{C}_1(s)}{G_u(s)[G_u(s)],
\]

(38)

where \(\hat{C}_1(s) \in RH_{\infty}\). From the assumption that the transfer function in (36) is stable and from (38), for \(s_1(i = 1, \cdots, n_1)\), which are the unstable poles of \(G_1(s)\) and for \(s_2(i = 1, \cdots, n_2)\), which are the unstable poles of \(G_2(s)\),

\[
1 - C_1(s_{1i}) \left(G_1(s_{1i})e^{-s_{1i}T_1} + G_2(s_{1i})e^{-s_{1i}T_2}\right) = 0 (i = 1, \cdots, n_1)
\]

(39)

and

\[
1 - C_1(s_{2i}) \left(G_1(s_{2i})e^{-s_{2i}T_1} + G_2(s_{2i})e^{-s_{2i}T_2}\right) = 0 (i = 1, \cdots, n_2)
\]

(40)

are satisfied. From Lemma 1, there exists \(G_u(s) \in \mathcal{U}\) satisfying

\[
1 - G_u(s_{1i}) \left(G_1(s_{1i})e^{-s_{1i}T_1} + G_2(s_{1i})e^{-s_{1i}T_2}\right) = 0 (i = 1, \cdots, n_1)
\]

(41)

and

\[
1 - G_u(s_{2i}) \left(G_1(s_{2i})e^{-s_{2i}T_1} + G_2(s_{2i})e^{-s_{2i}T_2}\right) = 0 (i = 1, \cdots, n_2).
\]

(42)

Note that the conditions in (41) and (42) are equivalent to (22) and (23), respectively. Using \(G_u(s) \in \mathcal{U}\) satisfying (41) and (42), \(C_1(s)\) is rewritten as

\[
C_1(s) = G_u(s) \left(1 + \frac{C_1(s) - G_u(s)}{G_u(s)}\right).
\]

(43)

Because \(G_u(s) \in \mathcal{U}\) and \(C_1(s) \in RH_{\infty}\), \((C_1(s) - G_u(s))/G_u(s)\) is stable. In addition, because \((C_1(s) - G_u(s))/G_u(s)\) takes the form

\[
\frac{C_1(s) - G_u(s)}{G_u(s)} = \frac{C_1(s)}{G_u(s)} - 1
\]

(44)

and both \(C_1(s)\) and \(1/G_u(s)\) are proper, \((C_1(s) - G_u(s))/G_u(s)\) is proper. Therefore, \((C_1(s) - G_u(s))/G_u(s) \in RH_{\infty}\).

From (39), (40), (41) and (42),

\[
C_1(s_{1i}) - G_u(s_{1i}) = 0 (i = 1, \cdots, n_1)
\]

(45)

and

\[
C_1(s_{2i}) - G_u(s_{2i}) = 0 (i = 1, \cdots, n_2)
\]

(46)

hold true. This implies that \(s_{1i}(i = 1, \cdots, n_1)\), which are poles of \(G_u(s)\), are zeros of \(C_1(s) - G_u(s)\), because \(G_u(s) \in \mathcal{U}\) and \(C_1(s) \in RH_{\infty}\). In like manner, \(s_{2i}(i = 1, \cdots, n_2)\), which are poles of \(G_u(s)\), are zeros of \(C_1(s) - G_u(s)\), because \(G_u(s) \in \mathcal{U}\) and \(C_1(s) \in RH_{\infty}\). When we rewrite \((C_1(s) - G_u(s))/G_u(s)\) as

\[
\frac{C_1(s) - G_u(s)}{G_u(s)} = \frac{Q(s)}{G_u(s)[G_u(s)],
\]

then \(Q(s) \in RH_{\infty}\), because \(1/(G_u(s)G_u(s)) \in RH_{\infty}\). In this way, it is shown that if the controller \(C(s)\) in (1) makes the control system in Fig. 1 stable and makes the transfer function from \(r\) to \(y\) of the control system in Fig. 1 have a finite number of poles, then \(C(s)\) is written as (25).

Next, the sufficiency is shown. If \(C(s)\) takes the form (25), then the controller \(C(s)\) makes the control system in Fig. 1 stable and makes the transfer function from \(r\) to \(y\) of the control system in Fig. 1 have a finite number of poles. After simple manipulation, we have

\[
\frac{C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})} = \frac{G_u(s) \left(1 + \frac{Q(s)}{G_u(s)[G_u(s)],
\]

(47)

and

\[
G_u(s) \left(1 + \frac{Q(s)}{G_u(s)[G_u(s)],
\]

(48)
\[
\frac{C(s)}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}
= \frac{G_u(s)}{G_{u1}(s)G_{u2}(s)} \left(1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)}\right), \tag{49}
\]

\[
\frac{G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}}{1 + C(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}
= \left\{1 - \frac{G_u(s)}{G_{u2}(s)} \left(1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)}\right) G_{s1}(s)e^{-sT_1} - \frac{G_u(s)}{G_{u1}(s)} \left(1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)}\right) G_{s2}(s)e^{-sT_2}\right\}
\cdot (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \tag{50}
\]

rewritten as
\[
G_{u1}(s)G_{u2}(s) = \sum_{i=1}^{n_1} \frac{c_{1i}}{s - s_{1i}} + \sum_{i=1}^{n_2} \frac{c_{2i}}{s - s_{2i}} + \delta, \tag{52}
\]

\[
G_u(s) \left(1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)}\right) G_1(s)
= \sum_{i=1}^{n_1} \frac{c_{1i}e^{s_{1i}T_1}}{s - s_{1i}} + Q_1(s), \tag{53}
\]

and
\[
G_u(s) \left(1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)}\right) G_2(s)
= \sum_{i=1}^{n_2} \frac{c_{2i}e^{s_{2i}T_2}}{s - s_{2i}} + Q_2(s) \tag{54}
\]
respectively, where
\[
c_{1i} = (s - s_{1i}) G_{u1}(s)G_{u2}(s)|_{s=s_{1i}} \quad (i = 1, \ldots, n_1) \tag{55}
\]
and
\[
c_{2i} = (s - s_{2i}) G_{u1}(s)G_{u2}(s)|_{s=s_{2i}} \quad (i = 1, \ldots, n_2). \tag{56}
\]

Because \(G_u(s) \in U, Q(s) \in RH_{\infty}, G_{s1}(s) \in RH_{\infty}, G_{s2}(s) \in RH_{\infty}, 1/G_{u1}(s) \in RH_{\infty}, 1/G_{u2}(s) \in RH_{\infty}\), the transfer functions in (48), (49) and (51) are stable. If the transfer function in (50) is unstable, the unstable poles of the transfer function in (50) are unstable poles of \(G_1(s)\) or that of \(G_2(s)\). From the assumption that \(G_u(s)\) satisfies (22) and (23), the unstable poles of \(G_1(s)\) and \(G_2(s)\) are not the poles of the transfer function in (50). Therefore, the transfer function in (50) is stable. In addition, because the transfer function from \(r\) to \(y\) of the control system in Fig. 1 is given by (48) and \(G_u(s) \in U, Q(s) \in RH_{\infty}, G_{s1}(s) \in RH_{\infty}, G_{s2}(s) \in RH_{\infty}, 1/G_{u1}(s) \in RH_{\infty}\) and \(1/G_{u2}(s) \in RH_{\infty}\), the transfer function from \(r\) to \(y\) of the control system in Fig. 1 has a finite number of poles.

We have thus proved Theorem 2.

Note 2: \(G_u(s)\) satisfying (22) and (23) is obtained using the method given in the proof of Theorem 2.3.3 in [16].

Note 3: Note that Theorem 2 includes the result in [22]. This is confirmed as follows: Let \(G_2(s) = 0\) in Theorem 2. Then Theorem 2 is rewritten by Theorem 2 in [22].

The modified Smith predictor in (25) is explained based on the frequency domain. In the time domain, using the modified Smith predictor in (25), the control input \(u(t)\) is given as follows. From the assumption that the unstable poles of \(G_1(s)e^{-sT_1}\) and that of \(G_2(s)e^{-sT_2}\) are distinct, and from (22) and (23), \(G_{u1}(s)G_{u2}(s), G_u(s)(1 + Q(s)/(G_{u1}(s)G_{u2}(s)))G_1(s)\) and \(G_u(s)(1 + Q(s)/(G_{u1}(s)G_{u2}(s)))G_2(s)\) can be
domain as $C(s)$ in (25) is confirmed as follows: Taking the Laplace transformation of (57) yields

$$
\delta u(s) = Q_1(s)e^{-sT_1}u(s) + Q_2(s)e^{-sT_2}u(s) - \int_{-T_1}^0 \sum_{i=1}^{n_1} c_{1i}e^{-sT_1}u(s)e^{s\tau}d\tau
$$

$$
- \int_{-T_2}^0 \sum_{i=1}^{n_2} c_{2i}e^{-sT_2}u(s)e^{s\tau}d\tau + \bar{G}_u(s) \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) (r(s) - y(s))
$$

From (52), (53), (54), (58) and simple manipulation, we have

$$
u(s) = \bar{G}_u(s) \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) (r(s) - y(s))
$$

$$
= \left[ G_{u1}(s)G_{u2}(s) - \bar{G}_u(s) \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) \right] \cdot \left[ (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \right] (r(s) - y(s))
$$

$$
= \frac{G_u(s)}{1 - C_f(s)(G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2})}.
$$

From the above equation, we find that the control input $u(t)$ in (57) is written in the frequency domain as $C(s)$ in (25).

Next, we explain the control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors in (25). The transfer function from the reference input $r$ to the output $y$ of the control system in Fig. 1 is written as

$$
y = \frac{\bar{G}_u(s)}{G_{u1}(s)G_{u2}(s)} \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) \cdot \left[ (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \right] r.
$$

Therefore, when $G_1(s)$ or $G_2(s)$ has a pole at the origin, for the output $y$ to follow the step reference input $r = 1/s$ without steady state error,

1. when $G_1(s)$ has a pole at the origin

$$
\frac{\bar{G}_u(0)G_{u1}(0)}{G_{u2}(0)} = 1
$$

2. when $G_2(s)$ has a pole at the origin

$$
\frac{\bar{G}_u(0)G_{u2}(0)}{G_{u1}(0)} = 1
$$

must be satisfied. Because $\bar{G}_u(s) \in \mathcal{U}$ satisfies (22) and (23), (61) and (62) hold true. This implies that when $G_1(s)$ or $G_2(s)$ has a pole at the origin, the output $y$ follows the step reference input $r$ without steady state error, independent of $Q(s) \in \mathcal{RH}_\infty$ in (26). On the other hand, when $G_1(s)$ and $G_2(s)$ have no pole at the origin, for the output $y$ to follow the step reference input $r = 1/s$ without steady state error,

$$
\bar{G}_u(0) \left( 1 + \frac{Q(0)}{G_{u1}(0)G_{u2}(0)} \right) \cdot \frac{G_{u1}(0)}{G_{u2}(0) + G_{u2}(0)} = 1
$$

must hold.

The disturbance attenuation characteristics are as follows. The transfer function from the disturbance $d$ to the output $y$ of the control system in Fig. 1 is given by

$$
y = \left( 1 - \frac{\bar{G}_u(s)}{G_{u1}(s)G_{u2}(s)} \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) \cdot (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) \right) d.
$$

Therefore, to attenuate the step disturbance $d = 1/s$ effectively, $Q(s)$ must satisfy

$$
\bar{G}_u(0) \left( 1 + \frac{Q(0)}{G_{u1}(0)G_{u2}(0)} \right) \cdot \frac{G_{u1}(0)}{G_{u2}(0) + G_{u2}(0)} = 1.
$$

5. DESIGN METHOD OF MODIFIED SMITH PREDICTOR

From preceding section, we can design stabilizing modified Smith predictor for $G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}$ using the parameterization of all stabilizing modified Smith predictors. However using the parameterization of all stabilizing modified Smith predictors, it is difficult to specify input-output characteristic of control system in Fig. 1. In this section, in order to specify input-output characteristic, we present a design method of modified Smith predictor using the parameterization of all stabilizing modified Smith predictors and the model matching method.

The problem considered in this section is to find $Q(s)$ in (26) to make the transfer function from $r$ to $y$ in Fig. 1 be

$$
y = \frac{\bar{G}_u(s)}{G_{u1}(s)G_{u2}(s)} \left( 1 + \frac{Q(s)}{G_{u1}(s)G_{u2}(s)} \right) \cdot (G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2}) r
$$

$$
= (G_{m1}(s)e^{-sT_1} + G_{m2}(s)e^{-sT_2}) r,
$$

(66)
where \( G_{m1}(s) \in RH_{\infty} \) and \( G_{m2}(s) \in RH_{\infty} \) are models to have desirable input-output characteristic.

From (66), we have

\[
Q(s) = \left( \frac{G_{m1}(s)G_{u2}(s)}{G_u(s)G_{s1}(s)} - 1 \right) G_{u1}(s)G_{u2}(s) \tag{67}
\]

and

\[
Q(s) = \left( \frac{G_{m2}(s)G_{u1}(s)}{G_u(s)G_{u2}(s)} - 1 \right) G_{u1}(s)G_{u2}(s). \tag{68}
\]

Equation (67) and (68) imply that \( G_{m1}(s) \) and \( G_{m2}(s) \) must satisfy

\[
G_{m1}(s)G_2(s) = G_{m2}(s)G_1(s). \tag{69}
\]

Under the condition in (69), \( Q(s) \) to satisfy (66) exists and is given by (67).

From Theorem 2, \( Q(s) \) must be included in \( RH_{\infty} \). The condition that \( Q(s) \) written by (67) is included in \( RH\infty \) is related to \( G_{m1}(s) \) and \( G_{m2}(s) \). Next, we clarify the condition of \( G_{m1}(s) \) and \( G_{m2}(s) \) to make \( Q(s) \) written by (67) to be included in \( RH_{\infty} \).

In order for \( Q(s) \) written by (67) to be stable, \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) \) must be stable. Since \( \bar{G}_u(s) \in \mathcal{U} \) and \( G_{m1}(s) \) is unstable, if \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) \) is unstable, then the unstable poles of \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) \) are the unstable poles of \( G_{u2}(s) \) or the unstable zeros of \( G_{s1}(s) \). Therefore, if the zeros of \( G_{m1}(s) \) include the unstable poles of \( G_{u2}(s) \) and the unstable zeros of \( G_{s1}(s) \), then \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) \) is stable. In the following, for simplicity, the unstable zeros of \( G_1(s) \) and that of \( G_2(s) \) are assumed to be distinct. That is, when \( z_{i1} = 1, \ldots , m_1 \) denote unstable zeros of \( G_1(s) \), \( z_{i1} \neq z_{j1} \) \( (i \neq j) \), \( i = 1, \ldots , m_1; j = 1, \ldots , m_1 \). In like manner, when \( z_{i2} = 1, \ldots , m_2 \) denote unstable zeros of \( G_2(s) \), \( z_{i2} \neq z_{j2} \) \( (i \neq j) \), \( i = 1, \ldots , m_2; j = 1, \ldots , m_2 \). Then, the condition that \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) \) is stable is written by

\[
G_{m1}(s) = 0 \quad (i = 1, \ldots , n_2) \tag{70}
\]

and

\[
G_{m1}(s) = 0 \quad (i = 1, \ldots , m_1). \tag{71}
\]

Even if \( G_{m1}(s) \) satisfies (70) and (71), \( Q(s) \) written by (67) is not necessarily stable. If the unstable zeros of \( G_{m1}(s)G_{u2}(s)/(G_u(s)G_{s1}(s)) - 1 \) include the unstable poles of \( G_{s1}(s) \) and that of \( G_{u2}(s) \), then \( Q(s) \) written by (67) is stable. This expression is equivalent to

\[
G_{m1}(s) = \frac{G_u(s_1)G_{s1}(s_1)}{G_{u2}(s_1)} = e^{s_1T_1} \quad (i = 1, \ldots , n_1) \tag{72}
\]

and

\[
\lim_{s \to s_{2i}} \frac{G_{m1}(s)}{s - s_{2i}} = \frac{G_{u1}(s_2)G_{s1}(s_2)}{G_{u2}(s_2)e^{-s_{2i}T_2}} \lim_{s \to s_{2i}} \{(s - s_{2i})G_{u2}(s)\} \quad (i = 1, \ldots , n_2), \tag{73}
\]

because of (22) and (70). From above discussion, if \( G_{m1}(s) \) is settled to hold (70), (71), (72) and (73), then \( Q(s) \) written by (67) is stable. Next, we clarify the condition that \( Q(s) \) written by (67) is proper. Since \( G_u(s) \) and \( G_{u2}(s) \) are biproper, it is obvious that the necessary and sufficient condition that \( Q(s) \) written by (67) is proper is \( G_{m1}(s)/G_{s1}(s) \) is proper. Since the relative degree of \( G_{s1}(s) \) equals that of \( G_1(s) \), the necessary and sufficient condition that \( Q(s) \) written by (67) is proper is that the relative degree of \( G_{m1}(s) \) is greater than or equal to that of \( G_1(s) \). In summary, if \( G_{m1}(s) \) and \( G_{m2}(s) \) are set satisfying (69), (70), (71), (72) and (73) and the relative degree of \( G_{m1}(s) \) is greater than or equal to that of \( G_1(s) \), then \( Q(s) \) written by (67) is included in \( RH_{\infty} \), that is, (66) holds true.

6. THE PARAMETERIZATION FOR GENERAL CLASS OF TIME-DELAY PLANTS WITH MULTIPLE TIME-DELAYS

In this section, we briefly summarize the parameterization of all stabilizing modified Smith predictors for general class of time-delay plants with multiple time-delays.

Consider the control system in Fig. 2.

![Fig. 2: Feedback control system for \( \sum_{i=1}^{N} G_i(s)e^{-sT_i} \)](image)

\( \sum_{i=1}^{N} G_i(s)e^{-sT_i} \) is the single-input/single-output time-delay plant with time-delays \( T_i > 0 (i = 1, \ldots , N) \). \( C(s) \) is the controller, \( y \in R \) is the output, \( u \in R \) is the input, \( d \in R \) is the disturbance and \( r \in R \) is the reference input. We assume that \( G_1(s) (i = 1, \ldots , N) \) is coprime, that is \( G_1(s) (i = 1, \ldots , N) \) is controllable and observable and \( G_1(s) + \sum_{i=2}^{N} G_i(s)e^{-s(T_1-T_i)} = 0 \) is of minimum phase, that is, \( G_1(s) + \sum_{i=2}^{N} G_i(s)e^{-s(T_i-T_1)} \) has no zero in the closed right half plane. Note that from the assumption that \( G_i(s) \) is coprime and \( G_1(s) + \sum_{i=2}^{N} G_i(s)e^{-s(T_i-T_1)} \) is of minimum phase, \( \sum_{i=1}^{N} G_i(s)e^{-sT_i} \) is controllable and observable. Without loss of generality, \( T_i \neq T_j (i \neq j; i = 1, \ldots , N; j = 1, \ldots , N) \) is satisfied.
The parameterization of all stabilizing modified Smith predictors is summarized in the following theorem.

**Theorem 3:** It is assumed that \( G_1(s) + \sum_{i=2}^{N} G_i(s)e^{-sT_i} \) is of minimum phase. For simplicity, the unstable poles of \( G(s) \) and that of \( G_i(s) \) are assumed to be distinct. That is, when \( s_j(i = 1, \ldots, N; j = 1, \ldots, n_i) \) denote unstable poles of \( G(s) \), \( s_j \neq s_k(i \neq k; i = 1, \ldots, N; j = 1, \ldots, n_i; k = 1, \ldots, n_k) \). Under these assumptions, there exists \( \bar{C}(s) \) satisfying

\[
\bar{C}(s) = \prod_{i=1}^{N} G_{ui}(s_i)  \prod_{k=1}^{N} G_{uk}(s_{kj}) \prod_{k=1}^{N} s \frac{G_{si}(s_{ij})e^{-s_{ij}T_i}}{G_{si}(s_{ij}T_i)}.
\]

where \( G_{ui}(s) = \prod_{i=1}^{N} G_{ui}(s_i) \) is a stable function of \( G_i(s) \) that is factorized as

\[
G_i(s) = G_{ui}(s)G_{si}(s) \quad (i = 1, \ldots, N),
\]

where \( G_{si}(s) \) is the unstable biproper minimum phase function and \( G_{ui}(s) \) is the stable function. Using these functions, the parameterization of all stabilizing modified Smith predictors \( C(s) \) is written as

\[
C(s) = \frac{C_f(s)}{1 - \sum_{i=1}^{N} G_i(s)e^{-sT_i}}, \quad (76)
\]

where \( C_f(s) \) is given by

\[
C_f(s) = \frac{\bar{C}_s(s)}{\prod_{i=1}^{N} G_{us}(s)} \left( 1 + \sum_{i=1}^{N} Q(s) \prod_{i=1}^{N} G_{us}(s) \right), \quad (77)
\]

and \( Q(s) \in RH_\infty \) is any function.

**Proof:** Theorem 3 is proved using the same procedure of the proof of Theorem 2. Therefore, the detail is omitted.

**Note 4:** When the modified Smith predictor in (76) is used, the control input \( u(t) \) is obtained using the same procedure described in Section 4. From the same discussion in Section 4, we can see that obtained control input \( u(t) \) is realizable, that is the control system in Fig. 2 is causal.

### 7. NUMERICAL EXAMPLE

Consider the problem finding the parameterization of all stabilizing modified Smith predictors for the unstable plant \( \bar{G}_1(s)e^{-sT_1} + \bar{G}_2(s)e^{-sT_2} \) written by

\[
G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} = \frac{1}{s - 1} e^{-0.25s} + \frac{s + 1}{(s + 2)(s + 1)} e^{-0.5s}, \quad (78)
\]

where

\[
G_1(s) = \frac{1}{s - 1}, \quad (79)
\]

\[
G_2(s) = \frac{s + 1}{(s - 1)(s + 2)}, \quad (80)
\]

\[
T_1 = 0.25[sec] \text{ and } T_2 = 0.5[sec].
\]

\( G_1(s) \) and \( G_2(s) \) are factorized by (24) as

\[
G_{u1}(s) = \frac{s + 1}{s - 1}, \quad (81)
\]

\[
G_{s1}(s) = \frac{s}{s + 1}, \quad (82)
\]

\[
G_{u2}(s) = 1 \quad (83)
\]

and

\[
G_{s2}(s) = \frac{s + 1}{(s + 1)(s + 2). \quad (84)
\]

One of \( \bar{G}_u(s) \) in (26) satisfying (22) is given by

\[
\bar{G}_u(s) = \frac{s + 3}{s + 1} e^{0.25s}. \quad (85)
\]

From Theorem 2, the parameterization of all stabilizing modified Smith predictors for \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) in (78) is given by (25), where

\[
C_f(s) = \frac{(s - 1)(s + 3)e^{0.25}}{(s + 1)^2} \left( 1 + \frac{s - 1}{s + 1} Q(s) \right), \quad (86)
\]

and \( Q(s) \in RH_\infty \).

Next using the method described in Section 5, we design a stabilizing modified Smith predictor for \( G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} \) in (78). \( G_{m1}(s)e^{-sT_1} + G_{m2}(s)e^{-sT_2} \) that has desirable input-output characteristic and satisfies (69), (70), (71), (72) and (73) is settle by

\[
G_{m1}(s)e^{-sT_1} + G_{m2}(s)e^{-sT_2} = \frac{16.5362(s + 1.935)}{(s + 20)(s + 0.8)} e^{-0.25s} + \frac{8.2681(s - 1)(s + 1)(s + 1.935)}{(s + 20)(s + 2)(s + 0.8)(s + 0.5)} e^{-0.5s}, \quad (87)
\]

where

\[
G_{m1}(s) = \frac{16.5362(s + 1.935)}{(s + 20)(s + 0.8)}, \quad (88)
\]

and

\[
G_{m2}(s) = \frac{8.2681(s - 1)(s + 1)(s + 1.935)}{(s + 20)(s + 2)(s + 0.8)(s + 0.5)}. \quad (89)
\]
From (67), settling $Q(s) \in RH_{\infty}$ as

$$Q(s) = \frac{11.8784(s + 2.479)(s + 1)(s + 0.7837)}{(s + 20)(s + 3)(s + 0.8)}, \quad (90)$$

we have

$$C_f(s) = \frac{16.5362(s - 1)(s + 1.935)}{(s + 20)(s + 0.8)}. \quad (91)$$

The step response of the control system in Fig. 1 using $C_f(s)$ in (91) is shown in Fig. 3. Figure 3 shows that the control system in Fig. 1 is stable and the output $y$ follows the step reference input $r$ without steady state error.

When the disturbance $d(t) = 1$ exists, the response of the output $y$ is shown in Fig. 4. Figure 4 shows that the step disturbance is attenuated effectively.

In this way, we find that using the result in this paper, we can easily obtain the parameterization of all stabilizing modified Smith predictors for time-delay systems and design stabilizing modified Smith predictors for time-delay plants with multiple time-delays.

Next, when the time-delays $T_1$ and $T_2$ in (78) have small uncertainties, we account for ability of the proposed design method and behavior of the output $y$. When the real plant are written by

$$G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} = \frac{1}{s - 1}e^{-0.275s} + \frac{s + 1}{(2s + 1)(s + 2)}e^{-0.55s}, \quad (92)$$

which is different from (78), the step response of the control system in Fig. 1 using $C_f(s)$ in (91) is shown in Fig. 5. Figure 5 shows that the control system in Fig. 1 is stable and the output $y$ follows the step reference input $r$ without steady state error, even if the time-delays $T_1$ and $T_2$ have small uncertainties.

When the real plant are written by

$$G_1(s)e^{-sT_1} + G_2(s)e^{-sT_2} = \frac{1}{s - 1}e^{-0.225s} + \frac{s + 1}{(2s + 1)(s + 2)}e^{-0.45s}, \quad (93)$$

which is different from (78), the step response of the control system in Fig. 1 using $C_f(s)$ in (91) is shown in Fig. 6. Figure 6 shows that the control system in Fig. 1 is stable and the output $y$ follows the step reference input $r$ without steady state error, even if the time-delays $T_1$ and $T_2$ have small uncertainties.

In this way, we find that the proposed method can apply for time-delay systems including small uncertainties in time-delays.
8. CONCLUSION

In this paper, we proposed the parameterization of all stabilizing modified Smith predictors for minimum-phase time-delay plants with multiple time-delays. First, the parameterization of all stabilizing modified Smith predictors for stable plants with multiple time-delays was proposed. Next, we expanded the result of the parameterization for stable plants with multiple time-delays and proposed the parameterization of all stabilizing modified Smith predictors for unstable plants with multiple time-delays. The control characteristics of the control system using the parameterization of all stabilizing modified Smith predictors and a design method of \( Q(s) \) in (26) to specify input-output characteristics using model matching method were also given. Finally, a numerical example was presented to show the effectiveness of the proposed parameterization.

References

Kou Yamada was born in Akita, Japan, in 1964. He received the B.S. and M.S. degrees from Yamagata University, Yamagata, Japan, in 1987 and 1989, respectively, and the Dr. Eng. degree from Osaka University, Osaka, Japan in 1997. From 1991 to 2000, he was with the Department of Electrical and Information Engineering, Yamagata University, Yamagata, Japan, as a research associate. Since 2000, he has been an associate professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. His research interests include robust control, repetitive control, process control and control theory for inverse systems and infinite-dimensional systems. Dr. Yamada received 2005 Yokoyama Award in Science and Technology and The 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON2005) Best Paper Award.

Hiroshi Takenaga was born in Fukui, Japan, in 1984. He received the B.S. degree in Mechanical System Engineering from Gunma University, Gunma Japan, in 2006. He is currently M.S. candidate in Mechanical System Engineering at Gunma University. His research interests include process control and time-delay systems.